VOL.5 NO.2 September 2016 - ISSN 2233 – 1859

Operations Research Society in Bosnia and Herzegovina

Southeast Europe Journal of Soft Computing

Available online: http://scjournal.ius.edu.ba

IUS Soft Computing Research Group

Intergeneration Social Mobility as a Markov Process

Narela Spaseski, Indira Hasanović

International University of Sarajevo,
Faculty of Business and Administration,
Hrasnicka Cesta 15, Ilidža 71210 Sarajevo, Bosnia and Herzegovina

narela7@yahoo.com; indira.hasanovic@gmail.com

Article Info

Article history:
Article received on July 4, 2016
Received in revised form on August 26, 2016

Keywords:
Intergeneration mobility, regular Markov chain and income inequality.

Abstract

Intergeneration social mobility is an old concern in both sociology and economics and refers to a change in the status of family members from one generation to the next. In the line with Markov Chain Theory, in this paper, we provide estimates for intergenerational mobility, which is measured in terms of probabilities. We believe that the social mobility as many other natural and social science process can be represented by Markov matrices.

The results from this study show that after 7 generations the distribution has converged to its stationary point. Meaning, if there is no policy initiative to shift the intergeneration immobility; UK will remain with distribution showing inequality and different opportunities for the young generations depending on their parental background. In addition, over time the number of individual belonging to low-income class has increased, from 0.21 to 0.289. This implies that in UK, the income inequality has been increasing trough the period. The problem is formulated by using the Wolfram Mathematical Programming System.

1. INTRODUCTION

Intergeneration social mobility is an old concern in both sociology and economics and refers to a change in the status of family members from one generation to the next. It is often highlighted in the literature as an important goal for social policy, emphasizing its significance for economic and social well-being. Generally, low mobility is associated to higher inefficiency and higher injustice (see Comi 2004). Meaning, most talented individuals will not be allocated in the best position and initial positions decides your welfare, not the individual effort.

In this paper, we will focus on *How* social mobility can be modelled and measured?

According Checchi (1997) intergenerational social mobility is measured following two main methodologies. The first estimates the income elasticity of offspring with respect to parents' income. The second approach is based on the estimate of Markov Chain Model, where mobility is measured in terms of probability of offspring to better or worse their economic conditions with respect to the parents' one.

Because the estimates from the first approach can present difficulties in its interpretations we follow the second approach for the analysis of the social mobility across British generations. We assume that the social mobility as many other natural and social science process can be entirely represented by Markov matrices, which elements are transition probabilities (see Bartholomew 1973). The transition probabilities measure the probability of change in income classes occurring from one period to the next one.

2. MARKOV CHAIN MODEL

Application of Markov chains are quite common and have become a standard tool of decision making. According to Hogben L. (1987), a "Markov chain" is a random process described by a physical system that at any given time (t = 1, 2, 3...) occupies one of a finite number of states. At each time t the system moves from state j to state i with probability p_{ij} that does not depend on t. The numbers p_{ij} are called "transition probabilities". A notable feature of Markov chains is that they are historyless – the next state of the system depends only on the current state, not any prior states (see Hefferon 2003). Andrei Nikolaevich Kolmogorov in his seminal work: *"Foundations of the Theory of Probability",* remarked that "Historically, the independence of experiments and random variables represents the very mathematical concept that has given probability its peculiar stamp".

It was Prais (1955ba, 1955) who first applied Markov Chain Theory to measure social mobility. Significant empirical contribution could be found also in the work of Kemeny and Snell (1960, 1962) and Feller (1968), where Markov chain is used as a model for both intergenerational and intergenerational social mobility.

In this paper we use the special kind of stochastic process, called a Regular Markov Chain.

DEFINITION 1: A Markov process is a *regular chain* if some power of the transition matrix has only positive entries.

In particular, the Markov process is regular if all entries in the transition matrix $P = P1$ are positive. A Markov process is a regular one if there is some positive integer n, so that the process may be in anyone of the possible states n steps after starting, regardless of the initial state. The smallest n for which this is possible is the smallest positive integer n for which Pn has no zero entries.

If P is the transition matrix of a regular Markov chain, then it turns out that the powers of P approach a matrix V, all of those rows are the same. If **v** denotes the row vector formed from any of the rows of V, then it also happens that **v**P= **V**.

DEFINITION 2: A vector v is a *fixed point vector* of the matrix *P* if *P* = V. A Markov chain is said to be in *equilibrium* if the probability distribution at some step is given by a fixed point vector of the transition matrix.

We should also emphasize that equilibrium in this paper does not imply that there is no movement of individuals between stats. On contrary, the stochastic concept of equilibrium explicitly requires that individuals move in and out of each class. But on the average, forces acting to increase the number of individual movements are exactly counterbalanced by those tending to decrease it.

3. APPLICATION

3.1. Data

The actual data used is obtained on the basis of the ***Study of Intergenerational Changes in*** **Status** *by Glass and Hall*. Studying the social mobility in UK the researchers come up with the following transition matrix P:

		1	**2**	**3**
P=	**1**	0.65	0.28	0.07
	2	0.15	0.67	0.18
	3	0.12	0.36	0.52

Note: state 1 is lower-income class, state 2 is middle-income class and state 3 is upper-income class.

3.2. Numerical Results

In the line with Markov Chain Theory, in this section, we provide estimates for intergenerational mobility, which is measured in terms of probabilities. We believe that the social mobility in UK can be entirely represented by Regular Markov Matrix.

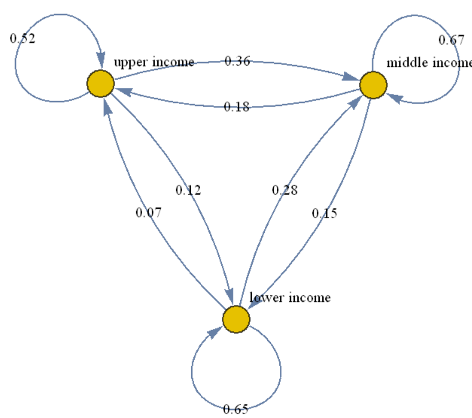

Figure 1. Transition diagram

The transition diagram given in Figure 1 shows the probability of change in income class from one generation to the next. If an individual is in state 2(middle-income class) then there is a probability of 0.15 that any offspring will be in the lower-income class, a probability of 0.67 that offspring will be in the middle-income class, and a probability of 0.18 that offspring will be in the upper-income class. But if we want to investigate the probability for changes in income class over two generations than we should use the tree diagram as shown in Figure 2. Since they record all possible outcomes in a clear and uncomplicated manner we can easily find out What is the probability that a grandchild will be in state 2(middle-income class) if a parent is currently in state 1(lower-income class)?

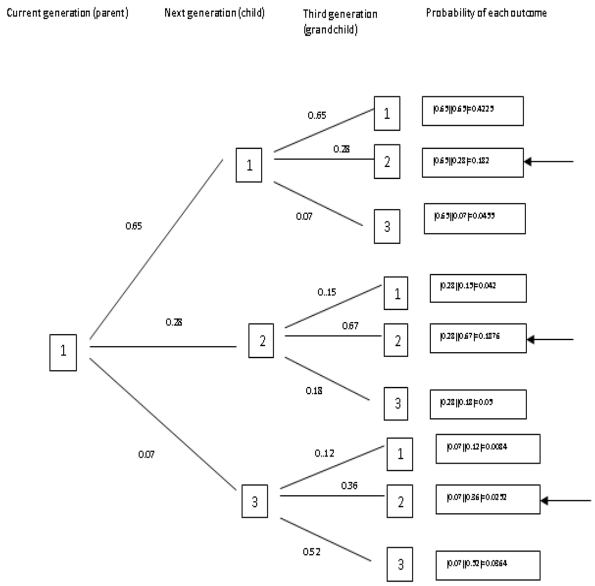

Figure 2. Tree diagram

As you can notice, the arrows point to the outcomes "grandchild in state 2". The probability that a parent in state 1 will have a grandchild in state 2 is given by sum of the probabilities indicated with arrows, or expressed as:

$$\begin{aligned}\text{Grandchild in state 2} &= p_{11}*p_{12}+p_{12}*p_{22}+p_{13}*p_{32}\\ &= 0.182+0.1876+0.0252\\ &= 0.3948 \text{ or } 39.48\%\end{aligned}$$

Here p_{ij}gives the probability that a person moves form state i to state j between generations (periods). For example, p_{13} represents the probability that a person in state 1 will have offspring in state 1, p_{11}=0.65. In general, this sum of products of probabilities is nothing more than one step in the process of multiplying matrix *P* by itself. Thus,

$$P^2= \begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 1 & 0.473 & 0.395 & 0.132 \\ 2 & 0.219 & 0.556 & 0.225 \\ 3 & 0.194 & 0.462 & 0.344 \end{array}$$

Note: The entry in row 1, column 2 in P^2 gives the probability that a person in state 1 will have a grandchild in state 2. Meaning that, a lower-income class person will have middle-income class offspring. This number, 0.39, is the result also found through using the tree diagram.
In the same way the matrix P^2 gives the probability of changes after two generations, the matrix $P^3=P*P^2$ gives the probability of changes after three generations.
For matrix P^3,

$$P^3= \begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 1 & 0.382 & 0.445 & 0.173 \\ 2 & 0.253 & 0.515 & 0.232 \\ 3 & 0.237 & 0.487 & 0.276 \end{array}$$

Matrix P^3 gives a probability of 0.445 that a person in state 1 will have great-grandchild in state 2. Also the probability that the person in state 1 will have great-grandchild in state 3 is 0.173. In order to develop a long-range prediction for the proportion of the population in each weight group, we should assign the initial distribution of people which can be written as probability vectors. If we suppose that the initial probability vector is Xo= [0.21 0.68 0.11], the distribution after one generation is:

Xo*P= [0.21 0.68 0.11]

Using this information, we can compute the distribution of weight group for two and more generations as illustrated in Table 1.

Table 1. Distribution of people in each social class, initial probability vector [0.21 0.68 0.11]

After Generation n	Low-income class	Middle-income class	Upper-income class
0	0.21	0.68	0.11
1	0.25	0.55	0.19
2	0.27	0.51	0.22
3	0.284	0.496	0.224
4	0.284	0.489	0.226
5	0.284	0.489	0.26
6	0.285	0.489	0.225
7	0.286	0.488	0.225
8	0.286	0.488	0.225

But what will happened if the initial probability vector is different from [0.21 0.68 0.11] ? If we suppose that [0.75 0.15 0.1] is used, then the results are given in Table 2.

Table 2. Distribution of people in each social class, initial probability vector [0.75 0.15 0.1]

After Generation n	Low-income class	Middle-income class	Upper-income class
0	0.75	0.15	0.1
1	0.522	0.347	0.132
2	0.348	0.459	0.192
3	0.348	0.459	0.192
4	0.318	0.475	0.207
5	0.303	0.482	0.215
6	0.295	0.485	0.222
7	0.291	0.487	0.222
8	0.289	0.488	0.224
9	0.289	0.488	0.224

The results in Table 2 are approaching the some numbers as with the initial probability vector [0.21 0.68 0.11]. In either case, the long-range trend is for about 48.8% of the people to be classified in the middle-income class. This example suggests that this long-range trend does not depend on the initial distribution.

But let us return, however, to the current situation in UK. As shown in Table 1, we can notice that after 7 generations the distribution has converged to its stationary point, [0.289 ;0.488 ; 0.224]. In addition, over time the number of individual belonging to low-income class has been increased, from 0.21 to 0.289. This implies that in UK, the income inequality has been increasing trough the period.

4. CONCLUSION

It is usual to think of Markov Chains as describing the trajectories of dynamic objects or people. The changes are not completely predictable, but rather are governed by probability distributions.

One of the main interested in intergenerational mobility is the concern with equality of opportunity for offspring's of different descent. Treating the intergenerational social mobility as a regular Markov Chain we find out that after 7 generations the distribution has converged to its stationary point (see Table 1). At this point forces acting to increase the number of individual movements are exactly counterbalanced by those tending to decrease it. If there is no policy initiative to shift the intergeneration immobility, UK will remain with distribution showing inequality and different opportunities for the young generations depending on their parental background.

Finally, with respect to the results, we find out that UK has rising income disparity - the number of individuals belonging to low-income class has been increased, from 0.21 to 0.289.

4. SELECTED EXTENSIONS

Intergeneration social mobility is a complex process therefore as interesting extensions we briefly described two of them which are of high importance in modelling complex system.

1. So-called adaptive Markov chains. These are systems in which the transition matrix is adjusted depending upon the entire history of the system or some statistical summary of that history.
2. Non-linear Markov Chain in which the distribution of X_n depends upon both X_{n-1} and its distribution, η_{n-1}. This is the evaluation of a Feynman-Kac system. Moreover, an excellent monograph on Feyman-Kac formulae and their mean field approximations has recently been written.

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