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Analytic Hierarchy Process (AHP) to Solve Complex Decision Problems

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ABSTRACT: In this thesis covers two different examples which we solved with Analytic Hierarchy Process (AHP).The Analytic Hierarchy Process was explained with details in this study. People encounter problems which are difficult to solve and understand. Decision making becomes more complex with apply common procedures without knowing any decision making application. AHP is the one of the application to use in decision analysis problem which is helping to change non-numerical judgments to convert in the system with numerical values for decision making process. It allows us to find out which alternative is the optimum as a result in the problem. In this study, 2 problems were solved with AHP. On the first example, it is considered for making decision to buy new phone. There are suggested 3 criteria and 4 alternatives.

1. INTRODUCTION

Industrial engineering applications for decision making process evaluate the different number of criteria. AHP (Analytic Hierarchy Process) is a widely used technique for decision making. AHP provides the best decision technique which has a big advantage for a non- numerical comparing criterion. It speeds upthe process and makes decision more systematic.

AHP has a high efficiency on multi criteria decision making problems. The AHP technique helps decision maker to set the priorities and make the best decision. It reduces complex criterion by converting problems to pairwise comparision (Triantaphyllou and Mann, 1995). To do so, AHP uses the fundamental scale of absolute numbers that validated by decision making experiments (Saaty, 1980, 1994).

AHP consists of a set of axioms and it is based on the weight matrices and their right eigenvectors. Merkin(Merkin,1979) and Saaty (Saaty,1980, 1994) described that AHP is generating true or approximating weights and structuring complexity, measuring on a ratio scale, and synthesizing.

As an integrative approach, the AHP supports four modes. These are absolute, distributive, ideal and super matrix modes for scaling weights to rank alternatives. In the absolute mode, when an alternative is added or deleted, rank reversal is not occurred and alternatives are rated one at a time. The second mode is distributive and it normalizes alternative weights in each criterion so they sum to one and this mode prevents ranking. In the ideal mode, the weight of each alternative is divided by the weight of the best one under criteria so it preserves rank. Final super matrix provides dependencies between different levels of a network. Recently, AHP was offered a DEAHP (Data Envelopment Analytic Hierarchy Process) which has no rank reversal but it suffers from rank reversal. (Ramanathan, 2006) Emrouznejad and Marra (Emrouznejad and Marra,2017) examine the development of AHP using social network analysis and scientometrics. Their analyses are based on 8441 papers published between 1979 and 2017. These are extracted from ISI Web of Science (WoS) database. The system investigates in three periods such as 1979–1990, 1991–2001 and 2002–2017. Researchers' aim is understanding the development of the AHP per year.

In ISI WoS academic database, papers are analyzed according to keywords. For instance, 'analytic hierarchy process'; 'AHP'; 'comparison matrix'; 'pair wise comparison matrix' and 'PCM'; 'matrix consistency' are used in this research. The data cover the periods from 1979 to 2017. The final research result includes 8441 published works: 3362 conference proceedings, 19 editorial pieces 211 articles and proceedings papers, 4721 papers and 128 other document types.

Figure 1 shows that the number of publications has increased over the last 10 years, with the highest numbers – more than 800 published works – in 2013 and 2015. The total sample includes papers published up to January 2017. (Emrouznejad and Marra, 2017)

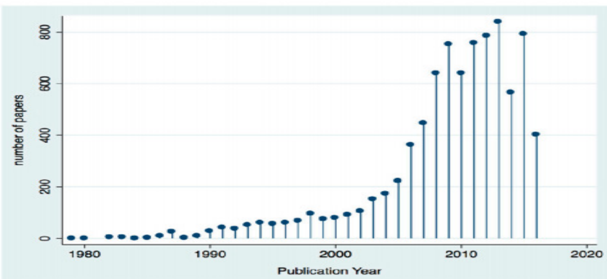

Figure 1. Number of publications related to the topic of AHP per year (1979-2017). (Emrouznejad and Marra, 2017)

Table 1. Top 10 most active journals in AHP. Emrouznejad and Marra (Emrouznejad and Marra, 2017) ranked journals according to the number of papers published inTable1. They obtain total local citation score (TLC) and total global citation score (TGCS).

Rating	Journals	Amount	TLC	TGCS
1	European Journal of Operational Research	214	1630	3012
2	Expert Systems with Applications	211	4387	6720
3	International Journal of Production Research	94	448	1163
4	Mathematical and Computer Modelling	73	508	819
5	International Journal of Production Economics	72	1243	2496
6	International Journal of Advanced Manufacturing Technology	66	260	591
7	Computers and industrial Engineering	59	305	614
8	Environmental Earth Science	54	63	93
9	Journal of Environmental Management	52	401	866
10	Journal of Operational Research Society	50	462	677

The first one refers to how many times the journal's papers included in this collection were cited by other papers in the collection; The second one refers to how many times the papers in the journals included in this collection were cited in the WoS(Web of Science) database. This score is calculated based on the Times Cited score retrieved from the WoS. Emrouznejad and Marra (Emrouznejad and Marra, 2017) described the top 10 most influential papers ranked by TLCS. The researchers also provide TGCS, which accounts for the impact of the paper within the entire ISI database.
It is ranked as follows:

1- Saaty (1990b), How to make a decision– The analytic hierarchy process. It is published in European Journal of Operational Research. TLCS is 642, TGCS is 836.

2- Saaty (1986), Axiomatic foundation of the analytic hierarchy process. It is published in Management Science. TLCS is 257, TGCS is 332.
3- Dyer (1990) Remarks on the analytic hierarchy process. It is published in Management Science. TLCS is 257, TGCS is 319.
4- Saaty (1994) How to make a decision – The analytic hierarchy process. It is published in Interfaces. TLCS is 201, TGCS is 277.
5- Harker and Vargas (1987) The theory of ratio scale estimation – Saaty Analytic hierarchy
Process. It is published in Management Science. TLCS is 193, TGCS is 209.
6- Forman and Peniwati (1998) Aggregating individual judgments and priorities with the
analytic hierarchy process. It is published in European Journal of Operational Research. TLCS is 184, TGCS is 173.
7- Saaty (1990a) An exposition of the AHP in reply to the paper remarks on the analytic hierarchy process. It is published in Management Science. TLCS is 172, TGCS is 190.
8- Crawford and Williams (1985) A note on the analysis of subjective judgement matrices. It is published in Journal of Mathematical Psychology. TLCS is 165, TGCS is 256.
9- Saaty and Vargas (1987) Uncertainty and rank order in the analytic hierarchy process. It is published in European Journal of Operational Research. TLCS is 150, TGCS is 173.
10- Ghodsypour and O'Brien (1998) A decision support system for supplier selection using an
integrated analytic hierarchy process and linear programming. It is published in International Journal of Production Economics. TLCS is 149, TGCS is 325.

2. BACKGROUND

For the first time in 1968, Albert and Myers found the Analytic Hierarchy Process on the theoretical part. Then, in 1977, Thomas Lorie Saaty found AHP. (Gülenç and Bilgin, 2010)

In 1980, Saaty introduced AHP to solve complex decision problems. AHP's validity is based on the actual applications and AHP results were accepted in these applications that used by the decision makers (Saaty 1994). It is an effective Multi Criteria Decision Making (MCDM) tool for setting priorities with calculated weighted values.

AHP is the most effective tool that used since the 1970's for decision making process. It provides setting priorities and making the best decision.

There are many AHP applications have been proposed in the literature.

Saaty (1977-1983) targets Eigenvector (EV) to analyze matrices of estimates. In this research, subjective matrices estimate the utility of one relative to another. Eigenvector

(EV) procedures is an effective tool for hierarchical problems.

Crawford and Williams (Crawford and Williams, 1985) applied different methods in hierarchical problems. The researchers presented the note on the analysis of subjective judgment matrices. The geometric mean (GM) vector (the logarithmic least squares method) can be applied to hierarchical problems in the same way. The difference is that the method is developed from statistical methods. For instance, it can be optimal if the judge's errors are multiplicative with a lognormal distribution. The GM provides the desirable attributes of the EV to apply in several areas. The Geometric Mean Scale, The Geometric Mean Vector and The Maximum Likelihood Estimator, Monte Carlo Comparison of Geometric Mean Vector and Eigen Vector Ratio Scales are explained in detail in this paper.

Analytic Hierarchy Process (AHP) has the set of criterion for corresponding hierarchic structure for a special case of priority setting in system. AHP's feedback allows for a wide class of dependencies. An operational basis for AHP derived a number of facts from these axioms providing. Saaty (Saaty, 1986) presented axiomatic foundation of the Analytic Hierarchy Process.

Harker and Vargas (Harker and Vargas, 1987) described the theory of ratio scale estimation – Saaty Analytic hierarchy process. The study is about major criticism with launching at the AHP

in our opinion, not valid. Illustrating through proof and through examples the validity or fallaciousness of the criticism.

Saaty and Vargas (Saaty and Vargas, 1987) explained uncertainty and rank order in the Analytic Hierarchy Process. The uncertainty alternatives are investigated such as uncertainty of the occurrence of events, uncertainty about the range of judgments and uncertainty to express preferences. In making comparison process, the uncertainty is measured by numerical values with each judgment and this method is applied to calculate the probability of an alternative or to rank project exchanges with other projects. The priority of each project with the probability can be obtained for final ranking. Stability of the eigenvector, Interval judgments, the probability of rank reversal hierarchy calculated and explained in this paper in order.

Saaty (Saaty, 1990b) summarized the philosophy and the principles of the AHP with general information, attributes, applications, and measurements. The fundamental information of AHP are described in this research. Also, the research has the highest TLCS and TGCS values in the most influential papers.

Dyer (Dyer, 1990) provides a brief review of several areas of operational difficulties with the AHP, and then focuses on the arbitrary rankings that occur when the principle of hierarchic composition is assumed. The AHP evaluates alternatives relative to a set of criteria have assumed this principle. The most important point is correcting this flaw with the concepts of the multi attribute utility theory.

AHP has been assumed that there is a unique way to deal with decision problems and traditional lines of utility theory largely reflected in the method. Implying in the case that the weights on two criteria are independent of the ratings used to measure performance. AHP has been arbitrary simply because there is no adhere to the axioms and outcomes of utility theory. (Saaty, 1990)

Saaty (Saaty, 1994) identified the ranking of importance, preference and likelihood with rating and comparison methods for solving decision problems.

AHP is often used in the group decision making process to achieve solution. Forman and Peniwati (Forman and Peniwati, 1998) developed the methodology about individual and group decision making. Aggregating Individual Judgments and Priorities with The Analytic Hierarchy Process is presented in 1998. The aggregation of individual judgments (AIJ) and the aggregation of individual priorities (AIP) methods are used to derive priorities for individuals.

Pareto principle, the use of arithmetic means, Ramanathan and Ganesh's methods are explained in this paper for group preferences.

Ghodsypour and O'Brien (Ghodsypour and O'Brien, 1998) explained a decision support system for supplier selection using an integrated analytic hierarchy process and linear programming because companies consider which supplier is the best and how much could be procured from each supplier. In supplier selection process, qualitative and quantitative factors are used to select best supplier. Mixed integer, multi-objective and goal programming are useful for this process but in this study, integrated analytic hierarchy process and linear programming is used with and without capacity constraints.

The number of complex issues described in recent years and researchers propose different AHP approaches.

Ramanathan and Ramanathan (Ramanathan and Ramanathan, 2010) used DEAHP for treating judgments qualitatively.

AHP sort approach is applied by Ishizaka, Pearman, and Nemery (Ishizaka, Pearman, and Nemery, 2012) and AHP-K-means algorithm – Vetois identified by Lolli, Ishizaka, and Gamberini (Lolli, Ishizaka, and Gamberini, 2014) for sorting problems.

Pairwise comparisons matrix (PCM) is another complex issue explained by Tomashevskii (2015), Dede et al. (2015, 2016) and Kułakowski (2015). Dede et al. (2015, 2016) presented the scheme which yields an estimate for the probability of rank reversal and test the applicability of this scheme under different conditions and a theoretical model for estimating the probability of the consequent rank reversal using the multivariate normal cumulative distribution function. The model of the expert estimation process is developed by Tsyganok, Kadenko, and Andriichuk (2012) for judgement scales.

A method for calculating the missing elements of an in complete matrix of PCM, by minimising a measure of global inconsistency is applied by Fedrizzi and Giove (2007), Sub-optimal heuristic algorithm is described by Siraj, Mikhailov, and Keane (2012a) for

consistency in Pairwise Comparisons Matrix (PCM). Consistency through optimization is another proposed approach that is presented by Benítez et al. (2012).

Principal eigenvector approach (Saaty, 2013) and Hadamard product induced bias matrix model (Kou, Ergu, and Shang, 2014) are also described for consistency in PCM.

New definition of Interval Multiplicative Comparison Matrices (IMCMs) incorporating consistency and indeterminacy levels of interval judgements (Li, Wang, and Tong, 2016) and new simulation algorithm designed for the AHP (Kazibudzki, 2016) are developed for consistency indices.

Indirect judgements (Siraj, Mikhailov, and Keane, 2012b), New method for deriving priority vectors that although based on the eigenvalue method is optimization -based (Grzybowski, 2013), Bayesian Prioritize Procedure (BPP) and Systemic Decision-Making in AHP (Salvador et al., 2014) and Hesitant AHP (Zhu and Xu, 2014) are described for prioritized method.

The proposed approaches for group decisions are two-dimensional Sammon's mapping; consensus convergence model (Srdjevic et al., 2013), precise consistency consensus matrix (Escobar, Aguarón, and Moreno-Jiménez, 2015), triangular FAHP to combine a triangular fuzzy weighted power geometric operator the recovery methods and extent analysis method effectively (Dong, Li, and Zhang, 2015), Group Euclidean distance, group minimum violations, and distance between weights for the purpose of evaluation (Grošelj et al., 2015),AHP-group decision-making model in a local context (a unique criterion) based on the individual selection of the numerical scale and prioritization method and a new individual consistency index(Dong and Cooper, 2016).

Inconsistency indices in PCM is explained by different researchers. The proposed approaches about this complex issue are investigation of the link between consensus and consistency; and between group decision and consistency, by defining general boundary properties for the inconsistency (Brunelli, Canal, and Fedrizzi, 2013), identification axiomatic properties of inconsistency indices (Brunelli and Fedrizzi, 2014), two new measures, termed congruence and consistency deadlock (Siraj, Mikhailov, and Keane, 2015) and new inconsistency index (Grzybowski, 2016).

2.1. Investigation Areas of AHP

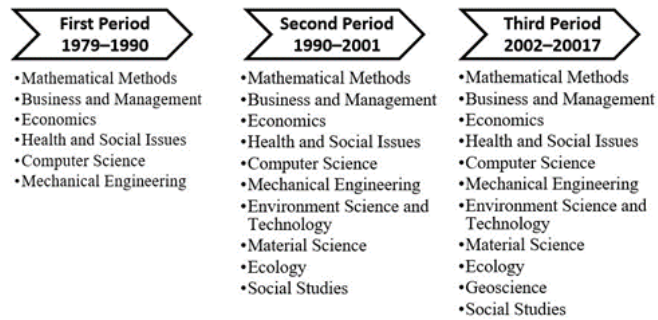

Figure 2. During the three sub-periods (1979–1990, 1990–2001 and 2002–20017). (Emrouznejad and Marra, 2017)

2.2. The Theoretical Basis of AHP

Belton and Gear (1983) first noticed about the validity of AHP. They found normalizing eigenvector weights of alternatives. These alternatives are used their max value rather than sum, which was also called B- G modified AHP. Then, Saaty and Vargas (1984) created a counterexample to them and they explained that B- G modified AHP is also subject to rank reversal. In 1985, Belton and Gear argued that their procedure was not understood. They insisted their research wouldn't result in any rank reversal if criterion's weight was changed. In 1989, referenced AHP is presented to avoid rank reversal by Schoner and Wedley. When an alternative is added or deleted, the research requires the modification or changing of criteria weights. Schoner et al., (1993) provided a method of normalization to the minimum and a linking pin AHP. In this research, criteria is chosen as the link for criteria comparisons and values are assigned in the linking cells. Barzilai et al., (1987) described that no normalization might prevent rank reversal. They also suggested a multiplicative aggregation rule to avoid rank reversal. It replaces normalized weight vectors with weight–ratio matrices. Barzilai and Lootsma, (1997) created a multiplicative AHP for preservation of ranking. Vargas Mianabadi and Afshar, (2007) suggested counter-example to show the invalidity of the multiplicative AHP. Then Triantaphyllou (2001) explained two new cases to demonstrate that the rank reversals don't occur with the multiplicative AHP, but they occur with the AHP.

Wang and Elhag developed an approach that the ranking among the alternatives would be preserved in the local priorities are not changed. (Arabameri, 2014)

2.3. Application Areas of AHP

Some of the areas where AHP is used in the diagram are shown;

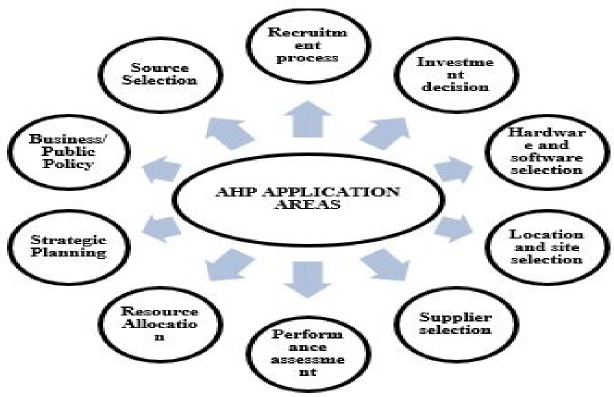

Figure 3. areas where AHP is used

3. METHODOLOGY OF AHP

Analytic Hierarchy Process is a hierarchical organization according to certain criteria, the weights of these criteria when evaluating an approach that enables comparing and sorting the alternatives according to the criteria. AHP has three basic principles that allows you to solve the decision problem. These principles are decomposition, comparative judgements and synthesis of priorities. With this method, a multi-criteria selection problem in the weights of the criteria the criteria for the purpose of determining the contribution of the movement of these values can be calculated and the best alternative can be selected.

3.1. The implementation of AHP

Step 1: The problem is defined. The goal, criteria and alternatives are expressed.

Step 2: Find relative importance of component weights in the hierarchy by using the scale 1-9 points for the purposes of the determination of the decision-making group by binary comparison Stage (Table-2).Comparison matrices are created. If there is n criteria to be evaluated, nxn comparison matrix A is created to determine the relative importance of the criterion i and the criterion j. Between matrix elements we have;

a_{ij} =1/a_{ji} and a_{ii}=1 (1)

Table 2. Table of relative scores.

a_{ij}	**Interpretation**
1	i and j are equally important
3	i is slightly more important than j
5	i is more important than j
7	i is strongly more important than j
9	i is absolutely more important than j
2,4,6,8	helping values for interpolated judgement

Step 3: Transformed binary comparisons of the priority vectors are calculated. We column normalize the matrix A as follows: LetS_jbe the sum of j^th column elements of the comparison matrix A. Then the elements of the column normalized matrix B are;

$$b_{ij} = \frac{a_{ij}}{s_j} \qquad \text{(i,j=1,2,…,n)} \qquad (2)$$

Finally, the components of the priority vector k is calculated by the formula;

$$k_i = \frac{\sum_{j=1}^{n} b_{ij}}{n} \qquad (3)$$

The priority vector k is indeed the approximately computed eigenvector of A, corresponding the largest eigenvalue of A.

Step 4: Involve the calculation of the Inconsistency Ratio (CR). Decision-making is made by the group for the ability to gauge the consistency of the comparison of the eigenvector method can be used.

For the calculation of λ_MAX,the approximate value of the largest eigenvalue of the matrix A, we multiply the priority vector k by the matrix A to obtain a vector d:

$$d = \begin{vmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{n1} & a_{n2} & a_{nn} \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \\ k_n \end{vmatrix} \qquad (4)$$

The largest eigenvalue λ_{MAX} of A is found by

$$\lambda_{MAX} = \frac{\sum_{i=1}^{n} E_i}{n} \qquad (5)$$

Where

$$E_i = \frac{d_i}{k_i} \qquad \text{(i=1,2,…,n)} \qquad (6)$$

Consistency Index is calculated by

$$CI = \frac{\lambda_{MAX} - n}{n-1} \qquad (7)$$

For each matrix of dimension $n \times n$, the Random Index (RI) is given by Saaty (Saaty, 1980) as in Table 3.;

Table 3. Values of the Random Index (RI) for small problems

n	2	3	4	5	6	7	8	9	10
RI	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

The value of the Inconsistency Ratio (CR) is obtained by dividing the value of Consistency Index (CI) with the value of Random Index (RI).

$$CR = \frac{CI}{RI} \qquad (8)$$

If the Inconsistency is smaller than 0.10, the comparison matrix indicate that decision makers are consistent, while the guess the element of the comparison matrix A. The value of CR greater than 0.10 that means that there is inconsistent comparisons or calculation error.

Until this step, any actions taken, decisions export the weights of the criteria that influence the solution of the problem can be determined. After this step, the analysis of alternatives could be performed by using comparison matrices, AHP different criteria weights as input for these methods of decision-making methods outside by passing through to be used may be provided.

Step 5: The criteria of decision alternatives within the scope of the bilateral comparisons are made by considering separately all the criteria. The number of criteria n matrix is created. The number of alternatives "m" is represented by each comparison matrix the size of m x mis required.

Step 6: The alternatives on the weighted score calculation is performed. The m x n comparison matrix is created by n unit of m x 1 size column vector from formed as a result of the analysis of alternatives. This matrix is multiplying with k column vector which is obtained as a result of criteria comparison. Result of calculation is giving a new vector. The elements of new vector are showing us the alternatives scores. Sum of the elements resulting must be 1 and the highest score (importance) is showing us which alternative is the best decision.

4. EXAMPLE

4.1.Buying a New Phone

An example which is buying a new phone, will be here described in order to clarify the mechanism of the AHP. m = 3 evaluation criteria are considered about performance, style and Battery Duration criteria. n=4 alternatives are evaluated about number of model phones.

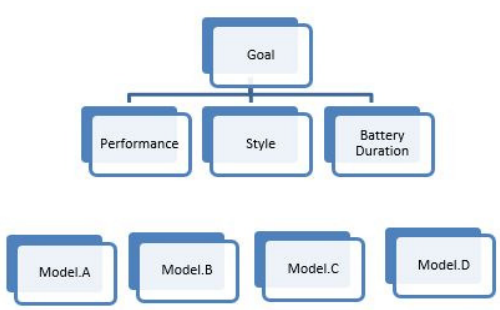

Figure.5 Buying a new phone hierarchy model

Each criterion is expressed by alternatives. The larger the value of the attribute, the better the performance of the option with respect to the corresponding criterion. The decision maker first builds the following pairwise comparison matrix for the three criteria:

1. Performance
2. Style
3. Battery Duration

We assumed the following judgments to determine the ranking of the criteria,

1) Performance is 2 times important than Battery Duration
2) Performance is 4 times important than Style
3) Battery Duration is 3 times important than Style

Table.4.Matrix and Result for pair wise comparison criteria

	Performance	Style	Battery	Priority
Performance	1	4.00	2.00	.558
Style	0.25	1	0.33	.122
Battery Dur.	0.50	3.00	1	.32

Inconsistency (CR): 0.019

Each of criteria has 4 alternatives which are computed individually. In this step, our judgment is made to determine model weights in each of criteria. Starting with performance of criteria;

-Model B is 2 times important than Model A
-Model A is 2 times important than Model C
-Model D is 6 times important than Model A
-Model B is 4 times important than Model C
-Model D is 2 times important than Model B
-Model D is 5 times important than Model C

Table.5. Matrix and Result for pair wise comparison altern-atives on performance criteria.

	Models				
Performance(.558)	A	B	C	D	Priority
Model A	1.00	0.50	2.00	0.17	.122
Model B	2.00	1.00	4.00	0.50	.266
Model C	0.50	0.25	1.00	0.20	.076
Model D	6.00	2.00	5.00	1.00	.536

Inconsistency (CR): .027

We assumed with the following judgments for criteria style;

-Model B is equal as Model A
-Model A is 3 times important than Model C
-Model D is 2 times important than Model A
-Model B is 3 times important than Model C
-Model B is 2 times important than Model D
-Model D is 4 times important than Model C

Table.6. Matrix and Result for pairwise comparison alternatives on style criteria.

	Models				
Style(.122)	A	B	C	D	Priority
Model A	1.00	1.00	3.00	0.50	.242
Model B	1.00	1.00	3.00	2.00	.355
Model C	0.33	0.33	1.00	0.25	.087
Model D	2.00	0.50	4.00	1.00	.316

Inconsistency (CR): .072

Last criteria on the example is about battery duration.

Battery Dur(.32)	Power(mAh)	Norm. Power
Model A	1600	.222
Model B	2000	.285
Model C	1200	.171
Model D	2200	.314

Table.7. Normalized power for each data, which was collected on model's company

Then we calculating the ranking score of models as in Table 8.

Table 8. Synthesis Model's weight's and score

	Weight	A	B	C	D
Performance	.588	.122	.266	.076	.536
Style	.122	.242	.355	.087	.316
Battery Dur.	.320	.222	.285	.171	.314
Synthesis		.168	.282	.110	.440

Introduce Prices of Four Models as Cost

As a final estimation, last ranking estimating result is the best options to choose it. But result of the ranking provides a logical framework for benefits determination to all alternatives. We can add up to one more extra criteria to compute score for decision makers. For example, ''Cost'' or ''Confort'' etc.

Table.9. Price table for all models, which are collected from online-marketing company

Alternatives	Cost ($)
Model.A	1.200
Model.B	500
Model.C	900
Model.D	1.800

All of alternatives have a cost value for influencing decision. As a cost to use normalized up to 1 and estimating normalized cost for calculating Benefit-Cost Ratio. This is also important for like a adjust second main criteria effects to decision result. First estimation gave us to Model D was the best solution with judging benefits. Then, cost effects the result, Model B is becoming the best decision for it.

There is a tool for comparing alternative's benefit-cost ratio which is Pareto frontier. Applied to Pareto frontier with these values are;

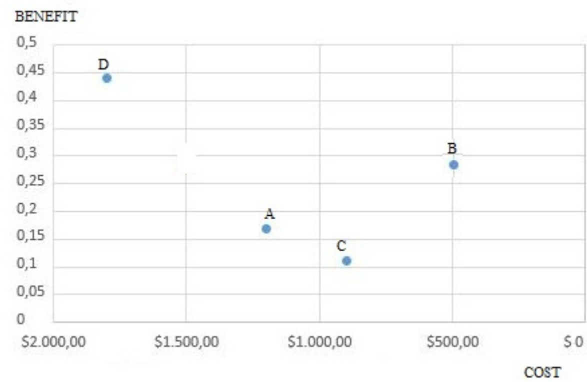

Figure 6. Pareto Diagram for benefit-cost ratio on first example

Our result of this example shows that benefit score is increasing with the cost, which means that the most beneficial alternative is the most expensive one except the alternative B. This proves us that B dominates A, and C in specified alternatives.

5. CONCLUSION

In this research, the Analytic Hierarchy Process application for solving decision problems is proposed. Before implementing analytic hierarchy process, investigation areas of AHP, the theoretical basis of AHP and application areas of AHP were explained in detail as a Literature Reviewing. As an output of this part, AHP was investigated in the three sub-periods (1979–1990, 1990–2001 and 2002–20017). The different approaches in the theoretical basis of AHP are explained in a timeline.

AHP is a decision problem application for solving more than one complex criteria. It allows you to model a hierarchical structure for showing a relation between decision makers in complex problems, the main objective of the problems, criteria, sub-criteria and alternatives. The most important feature of AHP is involving both objective and subjective thoughts to add on decision process. With another expression, AHP is a logical manner method where knowledge, experience thoughts and hunches of decision makers are combined.

On the model of AHP needs to be consistent for decision makers pairwise comparison. That's why, the method of Consistency Ratio was developed by Thomas L. Saaty. In the example, comparison matrices are consistent for each criteria and alternatives. Score of benefits is giving the best option to decide which phone is the best. But when we considered about price. We need to build up benefit-cost pareto diagram to check which alternative dominates. On pareto diagram, y axis shows about benefit score and x axis shows the cost. But the cost on the x axis must be opposite direction from the normal x axis numbers. Our result of this example shows that benefit score is increasing with the cost, which means that the most beneficial alternative is the most expensive one except the model B. This proves us that there B dominates alternatives C, and D.

For the further studies, the AHP applications would be analyzed deeply to define criteria and alternatives in order to prevent the any kind of losses. Apart from this, different integrated methodologies with AHP would be applied in the same examples and the results would be compared. Also, output of this study would be used to other researches.

6. REFERENCES

Arabameri, A. (2014). Application of the Analytic Hierarchy Process (AHP) for locating fire stations: Case study maku city. Merit Research Journal of Art, Social Science and Humanities, 2(1), 001-010.

Crawford, G., & Williams, C. (1985). A note on the analysis of subjective judgment matrices. Journal of mathematical psychology, 29(4), 387-405.

Dyer, J. S. (1990). Remarks on the analytic hierarchy process. Management science, 36(3), 249-258.

Emrouznejad, A., & Marra, M. (2017). The state of the art development of AHP (1979–2017): A literature review with a social network analysis. International Journal of Production Research, 55(22), 6653-6675.

Forman, E. H., & Gass, S. I. (2001). The Analytic Hierarchy Process—an exposition. Operations research, 49(4), 469-486.

Forman, E., & Peniwati, K. (1998). Aggregating individual judgments and priorities with the analytic hierarchy process. European journal of operational research, 108(1), 165-169.

Ghodsypour, S. H., & O'Brien, C. (1998). A decision support system for supplier selection using an integrated Analytic Hierarchy Process and linear programming. International journal of production economics, 56, 199-212.

Gülenç, İ. F., & Aydın Bilgin, G. (2010). Yatırım kararları için bir model önerisi: AHP yöntemi.

Harker, P. T., & Vargas, L. G. (1987). The theory of ratio scale estimation: Saaty's Analytic Hierarchy Process. Management science, 33(11), 1383-1403.

Kumar, S., Parashar, N., & Haleem, A. (2009). Analytical Hierarchy Process applied to vendor selection problem: Small scale, medium scale and large scale industries. Business Intelligence Journal, 2(2), 355-362.

Saaty, T. L. (1980). The Analytic Hierarchy Process, McGraw-Hill,

Saaty, T. L. (1986). Axiomatic foundation of the Analytic Hierarchy Process. Management science, 32(7), 841-855.

Saaty, T. L. (1990a). An exposition of the AHP in reply to the paper "remarks on the analytic hierarchy process". Management science, 36(3), 259-268.

Saaty, T. L. (1990b). How to make a decision: the Analytic Hierarchy Process. European journal of operational research, 48(1), 9-26.

Saaty, T. L. (1994). How to make a decision: The Analytic Hierarchy Process. Interfaces, 24(6), 19-43.

Saaty, T. L. (2008). Decision making with the Analytic Hierarchy Process. International journal of services sciences, 1(1), 83-98.

Saaty, T. L., & Vargas, L. G. (1987). Uncertainty and rank order in the Analytic Hierarchy Process. European Journal of Operational Research, 32(1), 107-117.

Triantaphyllou, E., & Mann, S. H. (1995). Using the Analytic Hierarchy Process for decision making in engineering applications: some challenges. International Journal of Industrial Engineering: Applications and Practice, 2(1), 35-44.