

## Total and Edge Domination on Anti Fuzzy Line Graph

<sup>1\*</sup> P. Kousalya, <sup>1</sup>Dr.C.Radhika and <sup>1</sup>Dr.N.Sathya Seelan

<sup>1</sup>Faculty of Mathematics,  
VET Institute of Arts and Science (co - ed),  
Thindal, Erode, Tamilnadu,

<sup>1</sup>Faculty of Mathematics,  
Thiru Kolanjiappar Government Arts College (Grade – I)  
Vridhachalam, Tamilnadu

\*Corresponding Author: [ramya8848@gmail.com](mailto:ramya8848@gmail.com) / [kousalyap@vetias.ac.in](mailto:kousalyap@vetias.ac.in)

---

### Article Info

#### Article history:

Article received on 9 10 2023

Received in revised form 01 11 2023

**ABSTRACT:** In this paper, the total dominating set, edge dominating set and domination number (TDN & EDN) for anti fuzzy line graph is computed by maxima method algorithm (MMA). This calculation is executed using strong adjacency matrix of anti fuzzy graphs and its line graphs

#### Keywords:

Total Domination Number (TDN); Edge Domination Number (EDN); Maxima Method Algorithm (MMA); Strong Anti Fuzzy Graph (SAFG); Anti Fuzzy Line Graph (AFLG)

---

## 1. INTRODUCTION

Anti fuzzy graph theory is one of the growing area in the field of uncertainty from the last decades. The motive of fuzzy graph theory is to reach the solution for the complexity problems within the nodes and its relation. Anti fuzzy graph theory will be the right solution when the ambiguity occurs on the relation between the nodes. Study of domination on graph plays a vital role in anti fuzzy graph. Muhammad Akram [4] introduced the concept of anti fuzzy structure in graphs. R. Seethalakshmi and R.B. Gnanajothi [5] introduced the definition of anti fuzzy graph. R.Muthuraj and A.Sasireka[10 - 13] illustrated some operations and domination algorithm on anti fuzzy graphs.

Anti fuzzy line graph is the anti fuzzy graph which is isomorphic to its corresponding AFG. [8] Toto Nusantara, Desi Rahmadani, Amianto, Yuliana

Trisanti and Abdhullah Bin Gani, studied the concept of AFLG. A.Somasundaram and S.Somasundaram[9] introduced total domination in fuzzy graphs using effective edges. In this paper, the total & edge domination set, TDN and EDN for the anti fuzzy line graph from strong adjacency matrix by using maxima method algorithm (MMA). The AFLG is derived from the base AFG. It is considered as the undirected AFG with the edges having reflexive and symmetric relation.

## 2. PRELIMINARIES

This topic includes some essential definitions to derive the outcome of this paper. This paper considers an undirected simple and connected graph.  $G(\sigma, \mu)$  is a fuzzy graph and  $GA(\sigma, \mu)$  is noted as anti fuzzy graph with underlying set  $S$ . Where,  $S$  is a fuzzy subset of non empty set. Notations and definitions are followed as in [13-17].

**1.1. Definition 2.1**

A Fuzzy Graph  $G(\sigma, \mu)$  consist of vertices  $(\sigma: S \rightarrow [0,1])$  and edges  $(\mu: S \times S \rightarrow [0,1])$  such that  $\mu(x,y) \leq \min(\sigma(x), \sigma(y))$  for all  $x$  and  $y$  in  $S$ .

**1.2. Definition 2.2**

$G_A(\sigma, \mu)$  is the Anti Fuzzy Graph which consist of vertices  $(\sigma: S \rightarrow [0,1])$  and edges  $(\mu: S \times S \rightarrow [0,1])$  such that  $\mu(x,y) \geq \max(\sigma(x), \sigma(y))$  for all  $x$  and  $y$  in  $S$ .

**1.3. Definition 2.3**

Anti fuzzy graph  $G_A(\sigma, \mu)$  is strong if  $\mu(x,y) = \max(\sigma(x), \sigma(y)) \forall (x,y)$  in  $\mu$ .  $G_A(\sigma, \mu)$  is complete if  $\mu(x,y) = \max(\sigma(x), \sigma(y)) \forall (x,y)$  in  $\sigma$ .

**1.4. Definition 2.4**

Order of  $G_A(\sigma, \mu) = \sum d_{G_A}(\sigma(x))$ . Size of  $G_A(\sigma, \mu) = \sum(\mu(x,y)) \forall (x,y)$  in  $\sigma$ .

**1.5. Definition 2.5**

The degree of a vertex  $\sigma(x)$  of an anti fuzzy graph is denoted and defined by  $d_{G_A}(\sigma(x)) = \sum_{x \neq y} \mu(x,y)$ .

**1.6. Definition 2.6 [11]**

A set  $D \subseteq V(G_A)$  is said to be a dominating set of an anti fuzzy graph  $G_A$  if for every vertex 'y' in  $(G_A) \setminus D$  there exist 'x' in  $D$  such that 'y' is a strong neighborhood of 'x' with  $\mu(x,y) = \max(\sigma(x), \sigma(y))$  otherwise it dominates itself.

A dominating set  $D$  with minimum number of vertices is called a minimal dominating set if no proper subset of  $D$  is a dominating set.

The maximum fuzzy cardinality taken over all minimal dominating set in  $G_A$  is called a domination number of anti fuzzy graph  $G_A$  and is denoted by  $\gamma'(G_A)$  or  $\gamma'A$ . That is,  $|D| f = \sum y \in D \sigma(y)$ .

**1.7. Definition 2.7 [12]**

A set  $D \subseteq V(G_A)$  is said to be a total dominating set of an anti fuzzy graph  $G_A$  if for every vertex  $y \in V(G_A) \setminus D$  is adjacent to at least one strong neighborhood vertex in  $D$  and the induced sub graph of  $D$  has no isolated vertices. That is, A dominating set  $D$  of an anti fuzzy graph  $G_A$  is said to be a total dominating set of  $G_A$  if there exists a vertex in  $D$  is not isolate.

A total dominating set  $D$  of an anti fuzzy graph  $G_A$  with minimum number of vertices is called a minimal total dominating set of  $G_A$  if no proper subset of  $D$  is a dominating set.

The maximum fuzzy cardinality taken over all minimal total dominating set is called total domination number (TDN) of  $G_A$  and it is denoted by  $\gamma_t(G_A)$  or  $\gamma A_t$ .

**1.8. Definition 2.8 [12]**

$G_A$  is an anti fuzzy graph and  $x,y \in V(G_A)$ . If 'y' is said to be a support vertex to 'x' then 'y' is adjacent to at least one end vertex 'x' in  $G_A$ .

Every support vertices in  $G_A$  contained in total dominating set of  $G_A$ . It may dominate more than one vertex in  $V \setminus D$  and also in  $D$ .

Example 1 [11]

Consider the anti fuzzy graph  $G_A$  with  $\sigma = \{a/0.5, b/0.6, c/0.3, d/0.1, e/0.7\}$  and  $\mu = \{ab/0.5, bc/0.6, cd/0.3, de/0.7, ae/0.7, be/0.7\}$ .

From this graph, the minimal edge dominating set is  $\{de, be\}$ . The edge domination number  $\gamma'(G_A) = 1.4$ .

Example 2 [12]

Consider the anti fuzzy graph  $G_A$  with  $\sigma = \{a/0.4, b/0.4, c/0.6, d/0.7, e/0.8, f/0.6, g/0.3, h/0.5\}$  and  $\mu = \{af/0.6, ae/0.8, bc/0.6, be/0.8, ed/0.8, cd/0.7, gf/0.6, gh/0.5\}$ .

From this graph, the total dominating sets are  $\{d,e,f,g\}$ ,  $\{d,e,f,h\}$  and  $\{d,e,f,a\}$ . The total domination numbers are 2.6, 2.3 and 2.4 respectively. The minimal total dominating set of  $G_A$  is  $\{d,e,f,g\}$  and  $\gamma_t(G_A) = 2.6$ .

**1.9. Definition 2.9 [15]**

$G_A(\sigma, \mu)$  be any anti fuzzy graph. The line graph of  $G_A$  denoted by  $L(G_A) = (X,Z)$ , is the graph with the set of vertices  $X = \{\{e\} \cup \{uv\} / e \in E, u,v \in V, e = uv\}$  and the set of edges  $Z = \{S_e S_f / S_e \cap S_f \neq \emptyset, e, f \in E, e \neq f\}$ .

Where

$$S_e = \frac{\{\{e\} \cup \{ue\}\}}{e \in E}, E, u, v \in V \tag{1}$$

Let  $(\sigma, \mu)$  be an anti fuzzy sub graph  $G_A$ . Define the anti fuzzy subset  $(\lambda, \omega)$  of  $(X,Z)$  respectively as follows,

$$\forall S_e = e \in Z, \lambda(S_e) = \mu(e) \tag{2}$$

$$\forall S_e S_f \in W, \omega(S_e S_f) = \max\{\mu(e), \mu(f)\} \tag{3}$$

$(\lambda, \omega)$  is an anti fuzzy sub graph of  $L(G_A)$ , called the anti fuzzy line graph corresponding to  $(\sigma, \mu)$ .

**3. MATRIX REPRESENTATION OF  $G_A(\sigma, \mu)$**

This topic includes the matrix representation of anti fuzzy graphs. General forms of Adjacency, incidence matrix are defined. Adjacency fuzzy matrix, strong adjacency matrix and edge adjacency matrix for AFG are given below.

**1.10. Definition 3.1**

Let anti fuzzy graph  $G_A(\sigma, \mu)$  be a  $(p,q)$  graph with  $\sigma = \{a_1, a_2, \dots, a_n\}$ . Consider the  $p \times p$  matrix  $A = [a_{ij}]$ .

$$a_{ij} = \begin{cases} \mu_{ij}, & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$\mu_{ij}(x,y) \geq \max(\sigma_i(x), \sigma_j(y))$  for all  $x$  and  $y$  in  $S$ .

The matrix  $A = [a_{ij}]$  of above representation is called adjacency matrix of  $G_A(\sigma, \mu)$ .

**1.11. Definition 3.2**

Let anti fuzzy graph  $G_A(\sigma, \mu)$  be a  $(p, q)$  graph with  $\sigma = \{ \sigma_1, \sigma_2, \dots, \sigma_n \}$  and  $\mu = \{ \mu_1, \mu_2, \dots, \mu_n \}$  Consider the  $p \times p$  matrix  $B = [b_{ij}]$

$$b_{ij} = \begin{cases} \mu_{ij}, & \text{if } \sigma_i \text{ is incident with } \mu_j \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$\mu_{ij}(x,y) \geq \max(\sigma_i(x), \sigma_j(y))$  for all  $x$  and  $y$  in  $S$ .

The matrix  $B = [b_{ij}]$  of above representation is called incidence matrix of  $G_A(\sigma, \mu)$ .

**1.12. Definition 3.3 [11]**

The edge-weighted, edge-adjacency matrix denoted by  ${}^{ew}A$  is a square unsymmetrical  $E \times E$  matrix defined as,

$$[{}^{ew}A]_{ij} = \begin{cases} 1, & \text{if the edges } i \text{ and } j \text{ are incident} \\ k, & \text{if the edges } i - j \text{ is weighted} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

**1.13. Definition 3.4 [11]**

An anti fuzzy graph  $GA$  with the fuzzy relation  $\mu$  to be reflexive and symmetric is completely determined by the adjacency fuzzy matrix and it is denoted by  $eA\mu$ .

$${}^eA\mu = \begin{cases} \mu(e_j), & \text{for } i \neq j \text{ and } e_j \text{ is adjacent to } e_i \\ \mu(e_i), & \text{for } i = j \end{cases} \quad (7)$$

**1.14. Definition 3.5[11]**

$GA$  be a simple connected anti fuzzy graph, the strong adjacency matrix  $(eA\mu)'$

is defined as,

$${}^eA\mu = \begin{cases} \mu(e_j), & \text{for } i \neq j \text{ and } e_j \text{ is adjacent to } e_i \\ \mu(e_i), & \text{for } i = j \end{cases} \quad (8)$$

**Example 3**

Consider the anti fuzzy graph  $G_A(\sigma, \mu)$ . It consists of  $\sigma = \{a/0.7, b/0.2, c/0.4, d/0.6\}$  and  $\mu = \{ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6\}$

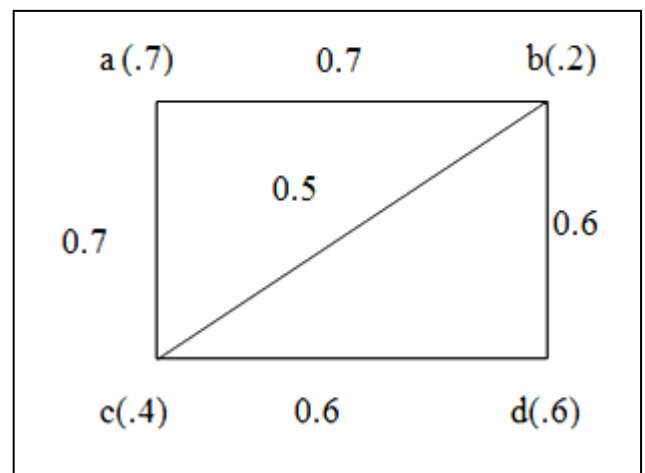


Figure 1: Anti Fuzzy Graph  $G_A$

The strong edge adjacency matrix  $({}^eA\mu)'$  is computed by considering the neighborhood weighted edges. It is given below.

|    |    |    |    |    |    |
|----|----|----|----|----|----|
|    | ab | ac | cd | cb | bd |
| ab | .7 | .7 | 0  | 0  | 0  |
| ac | .7 | .7 | 0  | 0  | 0  |
| cd | 0  | .7 | .6 | 0  | 0  |
| cb | .7 | .7 | 0  | .5 | 0  |
| bd | .7 | 0  | 0  | 0  | .6 |

Figure 2: Strong Edge Adjacency Matrix of  $G_A$

**4. ANTI FUZZY LINE GRAPH OF  $G_A(\sigma, \mu)$**

In this topic, the anti fuzzy line graph  $L(G_A)$  and its strong adjacency matrix  $L({}^eA\mu)'$  are evaluated from given anti fuzzy graph  $G_A$ .

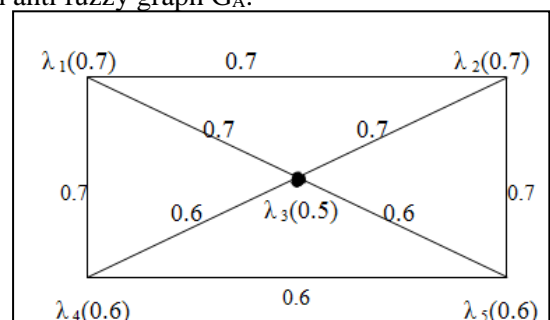


Figure 3: Anti Fuzzy Line Graph  $L(G_A):(\lambda,\omega)$

From example -2, the line graph of  $G_A$  is computed from figure – 1. It consists of  $(\lambda,\omega)$  such that,

$$\lambda = \{ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6\} \text{ and } \omega = \{\lambda_{12} / 0.7, \lambda_{14} / 0.7, \lambda_{13} / 0.7, \lambda_{23} / 0.7, \lambda_{34} / 0.6, \lambda_{25} / 0.7, \lambda_{35} / 0.6, \lambda_{45} / 0.6\}.$$

The strong edge adjacency matrix  $({}^eA_\mu)$  of  $L(G_A):(\lambda,\omega)$  is given below.

|                |                |                |                |                |                |                |                |                |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                | $\lambda_{12}$ | $\lambda_{13}$ | $\lambda_{14}$ | $\lambda_{23}$ | $\lambda_{25}$ | $\lambda_{34}$ | $\lambda_{35}$ | $\lambda_{45}$ |
| $\lambda_{12}$ | .7             | .7             | .7             | .7             | .7             | 0              | 0              | 0              |
| $\lambda_{13}$ | .7             | .7             | .7             | .7             | 0              | 0              | 0              | 0              |
| $\lambda_{14}$ | .7             | .7             | .7             | 0              | 0              | 0              | 0              | 0              |
| $\lambda_{23}$ | .7             | .7             | 0              | .7             | .7             | 0              | 0              | 0              |
| $\lambda_{25}$ | .7             | 0              | 0              | .7             | .7             | 0              | 0              | 0              |
| $\lambda_{34}$ | 0              | .7             | .7             | .7             | 0              | .6             | 0              | 0              |
| $\lambda_{35}$ | 0              | .7             | 0              | .7             | .7             | 0              | .6             | 0              |
| $\lambda_{45}$ | 0              | 0              | .7             | 0              | .7             | 0              | 0              | .6             |

Figure 4: Strong Edge Adjacency Matrix of  $L(G_A)$

### 1.15. Properties 4.1

Every  $L(G_A)$  is strong AFG.  $L({}^eA_\mu)$  is a square symmetric matrix with all entries are the strong edges of  $L(G_A)$ .

The main diagonal of  $L({}^eA_\mu) = \mu(e_i)$ .

Each  $e_i$  is specified by at least one fuzzy value corresponding to neighborhood to itself. That is, non zero row / column does not exist.

In  $L({}^eA_\mu)$ , a  $i^{\text{th}}$  row contains  $q$  number of entries then the corresponding edge is a neighbor to at least  $q$  vertices in  $L(G_A)$ .

$$\sum \mu(e_i) = \text{Size of } L[G_A(\lambda,\omega)] = \sum (\omega(x,y)) \quad \forall x,y \text{ in } \lambda \quad (9)$$

Sum of each row and column are equal. That is

$$\sum R(e_i) = \sum C(e_j) \quad (10)$$

$L({}^eA_\mu)$  is asymmetric matrix whose entries in main diagonal are  $\mu(e_i)$ .

In  $L({}^eA_\mu)$ , a  $i^{\text{th}}$  row contains  $q$  number of entries then the corresponding edge is a strong neighbor to  $q$  edges in  $L(G_A)$ . But the reverse part may differ.

## 5. EDGE DOMINATION ON ANTI FUZZY LINE GRAPH

The method of maxima algorithm is applied for computing edge and total domination number for anti fuzzy line graph. the EDN and TDN of Anti fuzzy graph and its line graphs are compared with each other.

### 1.16. Maxima Method Algorithm (MMA)

Construct the AFLG from the given anti fuzzy graph  $G_A$ .

Compute strong edge adjacency matrix  $L({}^eA_\mu)$ .

Set dominating set  $D = \phi$ .

Find  $\sum R(e_i)$ ,  $\sum C(e_i)$  and  $\sum(\text{main diagonal}) = S$ .

Select the maximum value in  $C(e_i)$ . If there exist a repeated value, choose an edge which have the maximum  $\mu(e_i)$  in main diagonal.

Choose the maximum edge in the selected column as the first element of dominating set  $D$ .

Delete the selected edge with corresponding row and column. In row, delete the edges which are incident with strong edges in  $D$ . take it as  $L({}^eA_\mu) \setminus 1$ .

If the reduced matrix  $L({}^eA_\mu) \setminus 1$  contains any row and column continue the process from step (4). Otherwise stop.

The resultant dominating set  $D$  of  $L(G_A)$  are the minimal edge dominating set of AFLG.

### 1.17. Edge Dominating Number (EDN) of $G_A$

Consider the anti fuzzy graph  $G_A(\sigma,\mu)$  from figure (1). It consists of  $\sigma = \{a/0.7, b/0.2, c/0.4, d/0.6\}$  and  $\mu = \{ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6\}$ . The strong edge adjacency matrix of  $G_A({}^eA_\mu)$  is given in Figure (2). The computation of dominating set is given below.

Set  $D = \phi$ .

Compute  $\sum R(e_i)$ ,  $\sum C(e_i)$  in  $({}^eA_\mu)$ .

$$\sum R(e_i) = \begin{matrix} ab \\ ac \\ cd \\ cb \\ bd \end{matrix} \begin{pmatrix} 1.4 \\ 1.4 \\ 1.3 \\ 1.9 \\ 1.3 \end{pmatrix}$$

$$\sum C(e_i) = \begin{pmatrix} ab & ac & cd & cb & bd \\ 2.8^* & 2.8 & 0.6 & 0.5 & 0.6 \end{pmatrix}$$

C(ab) has the maximum value. Select (ab) as the member of dominating set. That is, D = {ab}. Delete the selected column (ab) and the corresponding row (ab), (ac), (cb) and (bd).

The resultant matrix ( ${}^eA_\mu$ )'1

|          |     |
|----------|-----|
| ac       | cd  |
| [ 0.7* ] | 0.6 |

The above matrix has two columns then repeat the procedure. The edge (ac) has the maximum value. Then select (ac) as the member of D and delete the corresponding row of ( ${}^eA_\mu$ )'1.

The matrix has no column. Stop the process.

The minimal edge dominating set of anti fuzzy graph (figure -1) is D (G<sub>A</sub>) = {ab, ac}.

The EDN of anti fuzzy graph G<sub>A</sub>(σ, μ) is γ' = 1.4.

### 1.18. Edge Dominating Number (EDN) of L(G<sub>A</sub>)

Consider the example -2, the line graph of G<sub>A</sub> consists of (λ, ω) such that λ = {ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6} and ω = {λ<sub>12</sub> / 0.7, λ<sub>14</sub> / 0.7, λ<sub>13</sub> / 0.7, λ<sub>23</sub> / 0.7, λ<sub>34</sub> / 0.6, λ<sub>25</sub> / 0.7, λ<sub>35</sub> / 0.6, λ<sub>45</sub> / 0.6}. The strong edge adjacency matrix of L[G<sub>A</sub>] ( ${}^eA_\mu$ )' is given in Figure (4). The computation of dominating set is given below.

Set D = φ.

Compute  $\sum R(e_i)$ ,  $\sum C(e_i)$  in L( ${}^eA_\mu$ )'.

$$\sum R(e_i) = \begin{pmatrix} \lambda_{12} & 3.5 \\ \lambda_{13} & 2.8 \\ \lambda_{14} & 2.1 \\ \lambda_{23} & 2.8 \\ \lambda_{25} & 2.1 \\ \lambda_{34} & 2.7 \\ \lambda_{35} & 2.7 \\ \lambda_{45} & 2.0 \end{pmatrix}$$

$$\sum C(e_i) = \begin{pmatrix} \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{23} & \lambda_{25} & \lambda_{34} & \lambda_{35} & \lambda_{45} \\ 3.5 & 4.2^* & 3.5 & 4.2 & 3.5 & 0.6 & 0.6 & 0.6 \end{pmatrix}$$

Select λ<sub>13</sub> be the maximum value. D = {λ<sub>13</sub>}.

Delete the selected column λ<sub>13</sub> and its corresponding row λ<sub>12</sub>, λ<sub>13</sub>, λ<sub>14</sub>, λ<sub>23</sub>, λ<sub>34</sub> and λ<sub>35</sub>.

L( ${}^eA_\mu$ )'1 becomes,

|                 |                 |                 |                 |                 |                 |                 |                 |               |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|---------------|
|                 | λ <sub>12</sub> | λ <sub>14</sub> | λ <sub>23</sub> | λ <sub>25</sub> | λ <sub>34</sub> | λ <sub>35</sub> | λ <sub>45</sub> | $\sum R(e_i)$ |
| λ <sub>25</sub> | .7              | 0               | .7              | .7              | 0               | 0               | 0               | 2.1           |
| λ <sub>45</sub> | 0               | 0               | 0               | .7              | 0               | 0               | .6              | 2.0           |

$$\sum C(e_i) = \begin{pmatrix} 0.7 & - & 0.7 & 0.7 & 1.4 & - & - & 0.6 & 4.1 \\ & & & & * & & & & \end{pmatrix}$$

Select λ<sub>25</sub> is the member of dominating set and delete the column and its corresponding row. But the set D = {λ<sub>13</sub>, λ<sub>25</sub>} is isolated. Go to step 3.

From Step 3, select λ<sub>23</sub> be the element in D then delete the corresponding row λ<sub>12</sub>, λ<sub>13</sub>, λ<sub>23</sub>, λ<sub>25</sub>, λ<sub>34</sub> and λ<sub>35</sub>. The

|                 |                |                |                 |                |                 |                |                |                |               |
|-----------------|----------------|----------------|-----------------|----------------|-----------------|----------------|----------------|----------------|---------------|
|                 | λ <sub>1</sub> | λ <sub>1</sub> | λ <sub>14</sub> | λ <sub>2</sub> | λ <sub>25</sub> | λ <sub>3</sub> | λ <sub>3</sub> | λ <sub>4</sub> | $\sum R(e_i)$ |
|                 | 2              | 3              |                 | 2              | 3               | 4              | 5              | 5              |               |
| λ <sub>14</sub> | 7              | 7              | 7               | 0              | 0               | 0              | 0              | 0              | 2.1           |
| λ <sub>45</sub> | 0              | 0              | 7               | 0              | .7              | 0              | 0              | .6             | 2.0           |
| $\sum C(e_i)$   | 0.7            | 0.             | 1.4*            | 0              | 0.7             | -              | -              | 0.6            | 4.1           |
|                 |                | 7              |                 |                |                 |                |                |                |               |

resultant matrix

Therefore, D = {λ<sub>23</sub>, λ<sub>14</sub>} which is also isolated. Again go to step 3.

Select λ<sub>12</sub> be the next maximum element. Choose it is the member of dominating set D, delete the column and its corresponding row. D = {λ<sub>12</sub>}.

|                 |                |                |                |                |                |                |                |                |               |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---------------|
|                 | λ <sub>1</sub> | λ <sub>1</sub> | λ <sub>1</sub> | λ <sub>2</sub> | λ <sub>2</sub> | λ <sub>3</sub> | λ <sub>3</sub> | λ <sub>4</sub> | $\sum R(e_i)$ |
|                 | 2              | 3              | 4              | 3              | 5              | 4              | 5              | 5              |               |
| λ <sub>34</sub> | 0              | .7             | .7             | .7             | 0              | .6             | 0              | 0              | 2.7           |
| λ <sub>35</sub> | 0              | .7             | 0              | .7             | .7             | 0              | .6             | 0              | 2.7           |
| λ <sub>45</sub> | 0              | 0              | .7             | 0              | .7             | 0              | 0              | .6             | 2.0           |
| $\sum C(e_i)$   | -              | 1.4            | 1.4            | 1.4            | 1.4            | 0.6            | 0.6            | 0.6            | 7.4           |

Select λ<sub>13</sub> and delete the column with its corresponding row, then no column will exit in L( ${}^eA_\mu$ )'. D = {λ<sub>12</sub>, λ<sub>13</sub>, λ<sub>14</sub>}.

The different edge dominating sets are, D = {λ<sub>12</sub>, λ<sub>23</sub>, λ<sub>14</sub>}, {λ<sub>12</sub>, λ<sub>25</sub>, λ<sub>13</sub>} and {λ<sub>12</sub>, λ<sub>25</sub>, λ<sub>23</sub>}.

The EDN of anti fuzzy line graph L(G<sub>A</sub>(σ, μ)) is γ' = 2.1.

### 1.19. Alternate Procedure

From step 6, the set D = {λ<sub>13</sub>, λ<sub>25</sub>} is isolated then select the next maximum λ<sub>12</sub>. Because D = {ab, ac} = {λ<sub>1</sub>, λ<sub>2</sub>}. Obviously λ<sub>12</sub> be the one of dominating set element. Choose D = {λ<sub>12</sub>, λ<sub>13</sub>}.

|                | $\lambda_1$ | $\lambda_1$ | $\lambda_1$ | $\lambda_2$ | $\lambda_2$ | $\lambda_3$ | $\lambda_3$ | $\lambda_4$ | $\sum R(e_i)$ |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|---------------|
|                | 2           | 3           | 4           | 3           | 5           | 4           | 5           | 5           | $R(e_i)$      |
| $\lambda_{25}$ | .7          | 0           | 0           | .7          | .7          | 0           | 0           | 0           | 2.1           |
| $\lambda_{45}$ | 0           | 0           | .7          | 0           | .7          | 0           | 0           | .6          | 2.0           |
| $\sum C(e_i)$  | 0.7         | -           | 0.7         | 0.7         | 1.4         | -           | -           | 0.6         | 4.1           |

Delete the column and its corresponding row then  $\lambda_{45}$  will be the remaining. Choose  $\lambda_{14}$ . Hence  $D = D = \{\lambda_{12}, \lambda_{13}, \lambda_{14}\}$ .

The EDN of anti fuzzy line graph  $L(G_A(\sigma, \mu))$  is  $\gamma' = 2.1$ .

**1.20. Algorithm Modification**

By the above experimental results, this algorithm can be included the following steps. This algorithm will reduce the repetition of computation steps. From step 6,

Select the maximum value in  $C(e_i)$ . If there exist a repeated value, choose an edge which have the maximum  $\mu(e_i)$  in main diagonal.

If the elements of dominating set are isolated then choose the next maximum value in  $C(e_i)$  and proceed the same.

Obviously, this computation gives the minimal edge dominating set by deleting the subsequent maximum terms of  $C(e_i)$ .

**1.21. Total Dominating Number (TDN) of  $G_A$**

Consider the anti fuzzy graph  $G_A(\sigma, \mu)$  from figure (1). It consists of  $\sigma = \{a/0.7, b/0.2, c/0.4, d/0.6\}$  and  $\mu = \{ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6\}$ . The strong adjacency matrix of  $G_A (M\mu)'$  is given below.

$$(M\mu)' = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{pmatrix} . & . & . & 0 \\ . & . & 0 & 0 \\ . & 2 & 0 & 0 \\ . & 0 & 4 & 0 \\ 0 & 6 & 6 & 6 \end{pmatrix} \end{matrix}$$

Figure 5: Strong Adjacency Matrix of  $G_A$

**1.22. Total Dominating Number (TDN) of  $G_A$**

Compute  $\sum R(e_i), \sum C(e_i)$  in strong adjacency matrix  $(M\mu)'$ .

Reduce the matrix by deleting any row or column which are maximum.

|          | a   | b   | c   | d   | $\sum R(e_i)$ |      |      |
|----------|-----|-----|-----|-----|---------------|------|------|
| a        | .7  | .7  | .7  | 0   | 2.1*          | -    | -    |
| b        | .7  | .2  | 0   | 0   | 0.9           | 0.2  | 0.2  |
| c        | .7  | 0   | .4  | 0   | 1.1           | 0.4  | 0.4* |
| d        | 0   | .6  | .6  | .6  | 1.8           | 1.8* | -    |
| $\sum$   | 2.1 | 1.5 | 1.7 | 0.6 | 5.9           |      |      |
| $C(e_i)$ | -   | 0.8 | 1   | 0.6 |               | 2.4  |      |
|          | -   | 0.2 | 0.4 | -   |               |      | 0.6  |

The minimal total dominating set  $D = \{a,d,c\}$ . The total domination number TDN,  $\gamma_t(G_A) = 1.7$ .

**1.23. 5.7 Total Dominating Number (TDN) of  $L(G_A)$**

Consider the example -2, the line graph of  $G_A$  consists of  $(\lambda, \omega)$  such that  $\lambda = \{ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6\}$  and  $\omega = \{\lambda_{12}/0.7, \lambda_{14}/0.7, \lambda_{13}/0.7, \lambda_{23}/0.7, \lambda_{34}/0.6, \lambda_{25}/0.7, \lambda_{35}/0.6, \lambda_{45}/0.6\}$ . The dominating set is computed by the strong adjacency matrix given below. Continue the process till the row and columns are deleted.

$M\mu' =$

|             | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\sum R(e_i)$ |    |    |    |
|-------------|-------------|-------------|-------------|-------------|-------------|---------------|----|----|----|
| $\lambda_1$ | .7          | .7          | .7          | .7          | 0           | 2.8           | -  | -  | -  |
| $\lambda_2$ | .7          | .7          | .7          | 0           | .7          | 2.8           | 2. | -  | -  |
| $\lambda_3$ | .7          | .7          | .5          | 0           | 0           | 1.9           | 1. | 0. | 0. |
| $\lambda_4$ | .7          | 0           | 0           | .6          | 0           | 1.3           | 2  | 5  | 5  |
| $\lambda_5$ | 0           | .7          | 0           | 0           | .6          | 1.3           | 0. | 0. | -  |
| $\sum$      | 2.          | 2.          | 1.          | 1.          | 1.          | 10.           | 6  | 6  |    |
| $C(e_i)$    | 8           | 8           | 9           | 3           | 3           | 1             | 5. |    |    |
|             | -           | 2.          | 1.          | 0.          | 1.          |               | 2  |    |    |
|             | -           | -           | 0.          | 0.          | 0.          |               |    | 1. |    |
|             | -           | -           | 5           | 6           | 6           |               |    | 7  |    |
|             | -           | -           | 0.          | 0.          | 0.          |               |    |    | 1. |
|             | -           | -           | 5           | -           | 6           |               |    |    | 1  |

The minimal total dominating set  $D = \{\lambda_1, \lambda_2, \lambda_4, \lambda_5\}$ . The total domination number TDN,  $\gamma_t(G_A) = 2.6$ .

**6. CONCLUSION**

From figure (1),  $\mu = \{ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6\}$ . From figure (4),  $\lambda = \{ab/0.7, ac/0.7, cd/0.6, cb/0.5, bd/0.6\}$ . Then there exist a one-one

correspondence between the edge of  $G_A(\sigma, \mu)$  and the vertices of  $L(G_A):(\lambda, \omega)$ .

If any vertex is common in the dominating set then the corresponding edge is must be a member of dominating set in anti fuzzy line graph.

That is,  $ab = \lambda_1$  and  $ac = \lambda_2$ . Therefore  $\lambda_{12}$  be the obvious member of  $D(L(G_A))$ . If the procedure for searching the elements of dominating set is starts from this view, the other elements are identified easily.

| Results / Graph      | $G_A(\sigma, \mu)$ | $L(G_A):(\lambda, \omega)$                       |
|----------------------|--------------------|--|
| Edge Dominating Set  | {ab, ac}           | { $\lambda_{12}, \lambda_{13}, \lambda_{14}$ }   |
| EDN $\gamma'$        | 1.4                | 2.1  |
| Total Dominating Set | {a,d,c}            | { $\lambda_1, \lambda_2, \lambda_4, \lambda_5$ } |
| TDN $\gamma_t(G_A)$  | 1.7                | 2.6  |

**REFERENCES**

[1] L.A.Zadeh, Fuzzy Sets. Inform. Control.8(1965) 338-3. doi.org/10.1016/S00199958(65)90241-X.

[2] A. Rosenfeld, Fuzzy Graphs. In: Zadeh, L.A., Fu, K.S., Shimura, M (Eds.), "Fuzzy Sets and their Applications", Academic Press, New York, (1975). <https://doi.org/10.1016/B978-0-12-775260-0.50008-6>.

[3] Bhutani, K.R. "On Automorphism of Fuzzy Graphs", Pattern Recognition Letter 9 : 159-162,1989. [https://doi.org/10.1016/0167-8655\(89\)90049-4](https://doi.org/10.1016/0167-8655(89)90049-4).

[4] Muhammad Akram, "Anti Fuzzy Structures on Graphs", Middle East Journal of Scientific Research, Vol.1 11(12), (2012), pp.1641-1648. DOI:10.5829/idosi.mejsr.2012.11.12.131012.

[5] Seethalakshmi.R., and Gnanajothi. R.B., "Operations on Anti Fuzzy Graphs", Mathematical Sciences International Research Journal, Vol 5, Issue 2 (2016), pp.210-214.

[6] Mordeson, J.N. and P.S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs Physica Verlag, Heidelberg 1998:<https://doi.org/10.1007/978-3-7908-1854-3>.

[7] J.N.Mordeson, "Fuzzy Line Graphs", Pattern Recognition Letters, Volume 14, 1993, 381-384. [https://doi.org/10.1016/0167-8655\(93\)90115-T](https://doi.org/10.1016/0167-8655(93)90115-T).

[8] Toto Nusantara, Desi Rahmadani, Amianto, Yuliana Trisanti and Abdhullah Bin Gani, "Anti Fuzzy Line Graphs", Journal of Physics: Conference series, 1783 (2021). DOI: 10.1088/1742-6596/1783/1/012097.

[9] Somadsundaram. A., and Somadsundaram. S., "Domination in fuzzy graphs", Pattern Recognit. Let.,19(9)(1998),pp.787-791. [https://doi.org/10.1016/S0167-8655\(98\)00064-6](https://doi.org/10.1016/S0167-8655(98)00064-6).

[10] Muthuraj. R., and Sasireka. A., "On Anti fuzzy graphs", Advances in Fuzzy Mathematics, Vol. 12, No.5 (2017), pp. 1123 - 1135.

[11] Muthuraj. R., and Sasireka. A., "Domination on Anti Fuzzy Graph", international journal of Mathematical Archive, Vol.9, No. 5, 2018, pp. 82-92.

[12] Muthuraj. R.,and Sasireka. A., "Total domination on anti fuzzy graph", NTMSCI 6, No. 4, 28-39, 2018.<http://dx.doi.org/10.20852/ntmsci.2018.312>

[13] Muthuraj. R.,and Sasireka. A., "An Algorithm of Edge Domination Number on Anti Fuzzy Graph", Journal of Algebraic Statistics, Volume 13, No.2,2022,pp.121-25.<https://doi.org/10.5278/jas.v13i2.145>.

[14] Kousalya.P., Dr.Ganesan.V., and Tharani thiraviya.S., "Evaluation of Characteristics of Anti fuzzy graph", American International Journal of Research in Science, Technology, Engineering and Mathematics, Vol 1 Iss. 24,pp.22-25.

[15] Kousalya.P., Ganesan.V., and Anitha.L., "Anti Fuzzy Line Graph of Complementary Anti Fuzzy Graph" IJAAR,2022,Vol.9,Iss.3.

[16] Sathya Seelan.N., Kousalya.P., Ganesan.V., "Nordhaus-gaddum inequalities for anti fuzzy graph", Kongunadu Research Journal, 2022, Vol 9, Iss 1, pp.22-26.

[17] Kousalya.P., Ganesan.V., and Sathya Seelan.N., "Degree Boundaries and Model of Coin Splitting System in Anti Fuzzy Graph", Advances and Applications in Mathematical Sciences, 2022, Vol.21, Iss.3, pp. 1197-1208.